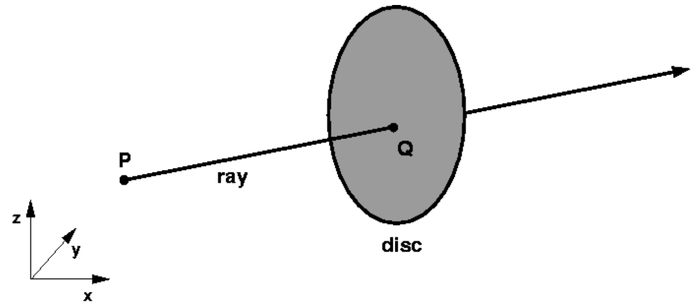
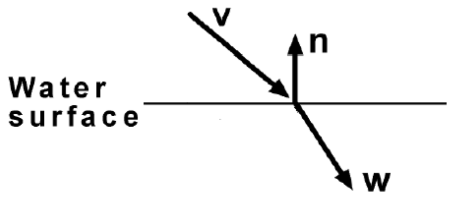
G-V Exercises

1. **Ray Tracing**

   (a) A disc is a planar face with a circular boundary. Describe an algorithm to find the intersection point Q of an arbitrary light ray from the camera position P with an arbitrary disc in 3D. Clearly state suitable input parameters to define both the ray and the disc.

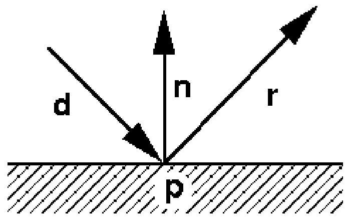


   (b) A light vector v passing from air into water is refracted on the water surface (see figure). Given v, and the *unit* normal n of the water surface, compute the resulting vector w. Assume that the projections of v and w onto n have the same length and that the projection of w onto the water surface has half the length of the projection of v onto the surface.



2. **Inverse Reflection**

   Let r be the direction vector of the perfect reflection of a light ray at a point p on a surface with normal n. Give the equations to compute the direction vector d of the original light ray (see figure below).



3. **Extended Light Sources**

   An extended light source is a light source which emits light from a surface, e.g. a sphere, rather than from a single point. With the help of a simple diagram explain how extended light sources create soft shadows. Briefly describe how you might extend the basic ray tracing algorithm to handle extended light sources to give soft shadows.

4. **Radiosity**

   Explain the radiosity equation and list all major steps required for the radiosity rendering technique.

G-V Exercise Solutions

1. **Ray Tracing**

   (a) INPUT:
   - Ray defined by camera position p and direction vector d
   - Disc defined by centre c, radius r and normal vector n

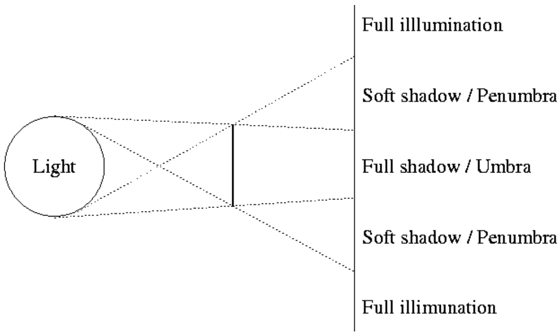
   I. Compute intersection point between ray and disc plane:
   Plane equation: $(x - c)^t n = 0$, $x \in \mathbb{R}^3$
   Ray parametrisation: $r(t) = p + td$, $t \geq 0$
   Intersection: $(p + td - c)^t n = 0$, $td^t n = (c - p)^t n$
   1. If $d^t n = 0$, return no intersection point
   2. $t = ((c - p)^t n)/d^t n$
   3. If $t < 0$, return no intersection point
      else $q = p + td$

   II. If $\|q - c\| \leq r$, return intersection point q
   else return no intersection point

   (b)
   - Let $m = (v^t n)n$
   - Let $e = v - m$
   - Then $w = m + e/2 = (v + m)/2 = (v + (v^t n)n)/2$

2. **Inverse Reflection**
   - Normalise: $N = n/\|n\|$, $R = r/\|r\|$.
   - Let $l = N^t RN$ be the projection of R onto N.
   - Then $2l = R - d$ or $d = 2l + R$.

3. **Extended Light Sources**
   - An extended light source may only be partially "visible" from a point. The areas where the light source is fully occluded are in the full shadow of the occluding surface; the areas where the light source if fully visible are fully lit; the areas in between are partially lit depending on how much of the light source is visible from the position.



   - For each extended light source cast multiple rays from the surface point to the extended light source to cover the whole light source surface.
   - The percentage of the rays that are not blocked by other surfaces gives the percentage of light reaching the point from the extended light source.

4. **Radiosity**
   - Radiosity models light as energy transfer between small surface patches based on thermal heat transfer. It is the rate at which energy leaves a surface patch which is equivalent to its brightness / intensity. Depending on the relative arrangement between two surface patches a certain fraction of the radiosity of one patch reaches the other patch. This gives the radiosity equation:

$$B_k = E_k + \rho_k \sum_j B_j F_{kj} \text{ for } k = 1, \ldots, n$$

B_k: radiosity/brightness of patch k
E_k: light/energy emitted by patch k
ρ_k: fraction of incoming light/energy reflected by patch k
F_{kj}: form factor between surface k and j indicating the fraction of light / energy leaving surface j and reaching surface k.

- Main radiosity processing steps:
  (a) Convert the parametric surfaces into a mesh consisting of small patches.
  (b) Compute the form factors F_{kj} for all patch pairs.
  (c) Set the emitted light E_k of each patch according to its material attributes.
  (d) Compute the energy intensity transfer by (approximately) solving the linear radiosity equation for red, green, and blue light.
  (e) Use the solutions B_k to shade the patches.