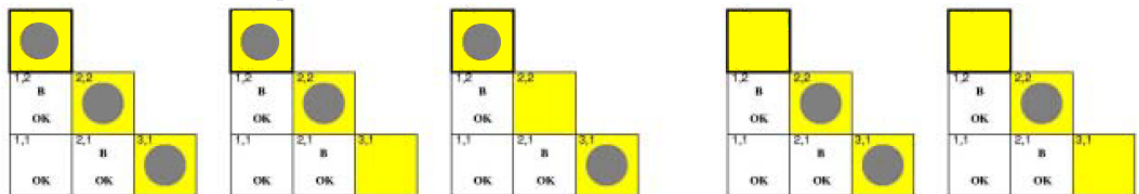


AI-III Solutions

1. Wumpus World

- Let $\mathbf{P}_{l,k}$ =true iff (l, k) contains pit. Let $\mathbf{B}_{l,k}$ =true iff (l, k) is breezy.
- We know
 $b : \neg b_{1,1} \wedge b_{1,2} \wedge b_{2,1}$
 $p : \neg p_{1,1} \wedge \neg p_{1,2} \wedge \neg p_{2,1}$
- Define for simplified notation:
Known := $(\mathbf{P}_{1,1}, \mathbf{P}_{1,2}, \mathbf{P}_{2,1})$
Fringe := $(\mathbf{P}_{1,3}, \mathbf{P}_{3,1})$
Other := $(\mathbf{P}_{1,4}, \dots, \mathbf{P}_{4,4})$
- b is conditional independent of **Other** given $\mathbf{P}_{2,2}$ and **Fringe**:
 $\mathbf{P}(b|\mathbf{P}_{2,2}, \mathbf{Known}, \mathbf{Other}, \mathbf{Fringe}) = \mathbf{P}(b|\mathbf{P}_{2,2}, \mathbf{Known}, \mathbf{Fringe})$
- $\mathbf{P}(\mathbf{P}_{2,2}|p, b)$
(inference by enumeration)
 $= \alpha \sum_{\text{fringe, other}} \mathbf{P}(b, p, \mathbf{P}_{2,2}, \text{fringe}, \text{other})$
(chain rule)
 $= \alpha \sum_{\text{fringe, other}} \mathbf{P}(b|p, \mathbf{P}_{2,2}, \text{fringe}, \text{other}) \mathbf{P}(p|\mathbf{P}_{2,2}, \text{fringe}, \text{other}) \mathbf{P}(\mathbf{P}_{2,2}|\text{fringe}, \text{other}) \mathbf{P}(\text{fringe}|\text{other}) \mathbf{P}(\text{other})$
(conditional independence and absolute independence assertions)
 $= \alpha \sum_{\text{fringe, other}} \mathbf{P}(b|p, \mathbf{P}_{2,2}, \text{fringe}) P(p) \mathbf{P}(\mathbf{P}_{2,2}) \mathbf{P}(\text{fringe}) \mathbf{P}(\text{other})$
(extract common factors)
 $= \alpha' \mathbf{P}(\mathbf{P}_{2,2}) \sum_{\text{fringe}} \mathbf{P}(b|p, \mathbf{P}_{2,2}, \text{fringe}) \mathbf{P}(\text{fringe}) \sum_{\text{other}} \mathbf{P}(\text{other})$
(sum over other is common factor, summing up to 1)
 $= \alpha' \mathbf{P}(\mathbf{P}_{2,2}) \sum_{\text{fringe}} \mathbf{P}(b|p, \mathbf{P}_{2,2}, \text{fringe}) \mathbf{P}(\text{fringe})$
(write out table)
 $= \alpha'' \langle P(p_{2,2}) \sum_{\text{fringe}} \mathbf{P}(b|p, p_{2,2}, \text{fringe}) \mathbf{P}(\text{fringe}),$
 $P(\neg p_{2,2}) \sum_{\text{fringe}} \mathbf{P}(b|p, \neg p_{2,2}, \text{fringe}) \mathbf{P}(\text{fringe}) \rangle$
 $= \alpha'' \langle 0.2 * (0.2 * 0.2 + 0.2 * 0.8 + 0.8 * 0.2 + 0.8 * 0.8), 0.8(0.2 * 0.2) \rangle \approx \langle 0.86, 0.14 \rangle$

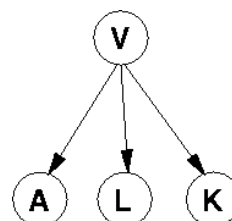
- Values for last step found from:



Note that we consider all options for $(1, 3)$ and $(3, 1)$ for each value of $\mathbf{P}_{2,2}$, giving probabilities in order from left to right as $0.2 * 0.2$ (case $p_{2,2}$), $0.2 * 0.8$ (case $p_{2,2}$), $0.2 * 0.2$ (case $\neg p_{2,2}$), $0.8 * 0.2$ (case $p_{2,2}$), $0.8 * 0.8$ (case $p_{2,2}$). Except for the last step the reasoning is the same as in the lecture. In this last step different squares / random variables are considered.

2. Bayesian Networks

- (a) • A, L and K are conditionally independent given V .
- Bayesian Network:



Correct as local semantics mean that children are conditionally independent of their non-descendants given their parents.

- (b)
 - $P(v|a, \neg l, \neg k) = \frac{P(a, \neg l, \neg k|v)}{P(a, \neg l, \neg k)}$
(Bayes' theorem)
 - $= \frac{P(a|v)P(\neg l|v)P(\neg k|v)P(v)}{P(a|v)P(\neg l|v)P(\neg k|v)P(v) + P(a|\neg v)P(\neg l|\neg v)P(\neg k|v)P(\neg v)}$
(global semantics of Bayesian network)
 - $= \frac{(.9)(.8)(.9)(.2)}{(.9)(.8)(.9)(.2) + (.2)(.2)(.2)(.8)}$
(values derived from table)
 - $= \frac{1296}{1360} \approx .9529$
 - In the table all entries must be between 0.0 and 1.0 and $P(v) + P(\neg v) = 1.0$.
- (c)
 - Experimentally $P(l|k) = .2$, $P(l|a, k) = .1$,
 - The network makes the assumption $P(L|K) = P(L|A, K)$
 - This is clearly not fulfilled. A suitable network would be a complete network with an additional link between A and L (either direction).
 - As we do not have any information about whether any of the e-mails contain a virus, it has no implication on the structure of the network from Question 3(a).

3. Diseases

- (a)
 - (1) is not asserted as `CommonCold` and `ExposureToCold` are linked by an arc.
 - (2) is not asserted by the structure of the network alone as arcs do not deny independence. One would need to know the conditional probability tables.
 - (3) is asserted as there is no arc
 - (4) is asserted due to local semantics of the network; `Vomit` and `SoreThroat` are conditionally independent.
 - (5) is asserted as `ExposureToOthers` and `ExposureToCold` are (absolutely) independent.
- (b)
 - We get the complete probability distribution from the global semantics of the Bayesian Belief Network:

$$P(x_1, \dots, x_n) = \prod_{l=1}^n P(x_l | \text{Parents}(x_l))$$

- So for the given network we insert the knowledge of all events with `Influenza = true` in the formula

$$\begin{aligned}
 &P(\text{ExposureToOthers})P(\text{ExposureToCold}) \\
 &\quad P(\text{CommonCold} | \text{ExposureToOthers}, \text{ExposureToCold}) \\
 &\quad P(\text{Influenza} | \text{ExposureToOthers}, \text{ExposureToCold}) \\
 &\quad P(\text{SoreThroat} | \text{CommonCold})P(\text{Fever} | \text{CommonCold}, \text{Influenza}) \\
 &\quad P(\text{Headache} | \text{CommonCold}, \text{Influenza})P(\text{Vomit} | \text{Influenza})
 \end{aligned}$$

- (c)
 - $P(\text{Fever} \wedge \text{Vomit} | \text{CommonCold}, \text{Influenza}, \text{ExposureToCold}) =$
 $P(\text{Fever} | \text{CommonCold}, \text{Influenza})$
 $P(\text{Vomit} | \text{Influenza})$
 - Due to local semantics in Bayesian Belief Networks `Fever` and `Vomit` are conditionally independent given knowledge of their parents.
- (d)
 - Conditional probability is defined by $P(a|b)P(b) = P(a \wedge b)$.
 - So we have $P(a \wedge b) = P(a|b)P(b) = P(b|a)P(a) = P(b \wedge a)$
 - which yields Bayes' rule: $P(b|a) = P(a|b)P(b)/P(a)$

4. Politics

- (a) (i) is not asserted as **H** and **S** are linked by an arc.
 (ii) is asserted, by the local semantics of Bayesian Networks: a node is conditionally independent of its non-descendants given its parents.
 (iii) is not asserted by the structure alone, because arcs do not deny independents (conditional probability tables can deny it, however).
- (b) $P(h, s, \neg p, \neg e) = P(h)P(s|h)P(\neg p|h, s)P(\neg e|\neg p) = 0.1 \times 0.3 \times 0.1 \times 0.9 = 0.00027$
- (c) One way is to use enumeration:

$$\mathbf{P}(\mathbf{E}|h) = \alpha P(h) \sum_s P(s|h) \sum_p P(p|h, s) \mathbf{P}(\mathbf{E}|p)$$

$$= \alpha 0.1 [0.3(0.9\langle 0.6, 0.4 \rangle + 0.1\langle 0.1, 0.9 \rangle) + 0.7(0.5\langle 0.6, 0.4 \rangle + 0.5\langle 0.1, 0.9 \rangle)] = \langle 0.41, 0.59 \rangle$$
- (d) Assume honesty does not influence fund-raising ability, but slickness does. Funds support advertising which increases popularity, but do not directly affect electability otherwise. So **L** should be a child of **S** and parent of **P**. We would need a conditional probability table $\mathbf{P}(\mathbf{L}|\mathbf{S})$ and an augmented conditional probability table $\mathbf{P}(\mathbf{P}|\mathbf{H}, \mathbf{S}, \mathbf{L})$.