AI-II Exercises

1. **Propositional Logic**

   Which of the following statements are true?

   (1) False $\models$ True

   (2) True $\models$ False

   (3) $(A \wedge B) \models (A \Leftrightarrow B)$

   (4) $(A \vee B) \wedge (\neg A \Rightarrow B)$ is satisfiable

2. **First-Order Logic I**

   (a) Translate each of the following sentences in English into first-order logic.

   A: Some cats like milk.
   B: Every cat has caught at least one mouse.
   C: There is a cat in Cardiff which has caught more mice than any cat in London.
   D: Any apartment in Cardiff has lower rent than some apartments in London.
   E: There is exactly one apartment in London with rent below £1000.
   F: If an apartment is more expensive than all apartments in Cardiff, it must be in London.

   (b) Translate the following sentences into good, natural English (without xs and ys!).

   i. $\exists x, y$ MailCarrier$(x) \wedge$ MailCarrier$(y) \wedge$ At(x, Cardiff) $\wedge$ At$(y,$ Cardiff$) \wedge x \neq y$.

   ii. $\forall x, y$ MailCarrier$(x) \wedge$MailCarrier$(y) \wedge$At$(x,$ Cardiff$) \wedge$At$(y,$ Cardiff$) \Rightarrow$ Knows(x, y).

   iii. $\exists d$ Dog$(d) \wedge [\forall c$ Cat$(c) \wedge (\exists b$ Bird$(b) \wedge$ Eats$(c, b)) \wedge$ Hates$(d, c)]$.

3. **First-Order Logic II**

   (a) Translate the following sentences into first-order logic
   A: Everyone's DNA is unique and is derived from their parents' DNA.
   B: Exactly one mouse is trapped.
   C: At most one cat fell into the pit.

   (b) Consider the following two suggested inference rules for propositional logic:

   $$\text{modus tollens: } \frac{P \Rightarrow Q, \neg Q}{\neg P}, \qquad \text{abduction: } \frac{P \Rightarrow Q, Q}{P}.$$

   Show that the modus tollens is sound and abduction is unsound. Despite abduction being unsound, in which situations may it still be suitable to apply?

4. **Cats and Fish**

   Cats eat everything they like and we know all cats like fish. Does Ziggy, the cat, eat fish?

   Use (a) forward chaining and (b) backward chaining to answer this simple query.

5. **Resolution in First-Order Logic**

   Consider the following knowledge base of two first-order logic sentences:

   A: $\forall x \, \forall y$ LostATrial$(x) \wedge$ Client$(y, x) \Rightarrow \neg$Happy(y).
   B: $\forall x$ SuccessfulLawyer$(x) \Rightarrow \exists y$ Client$(y, x) \wedge$ Happy(y).

   Use resolution to prove that this knowledge base entails the sentence

   C: $\forall x$ SuccessfulLawyer$(x) \Rightarrow \neg$LostATrial(x)

6. **Marcus hates Cæsar**

   Consider the following sentences:

   A: Marcus is a human.

   B: Marcus is a Pompeian.

   C: All Pompeians are Romans.

   D: Cæsar is a ruler.

   E: All Romans are either loyal to Cæsar or hate Cæsar.

   F: Everyone is loyal to someone.

   G: Humans only try to assassinate rulers they are not loyal to.

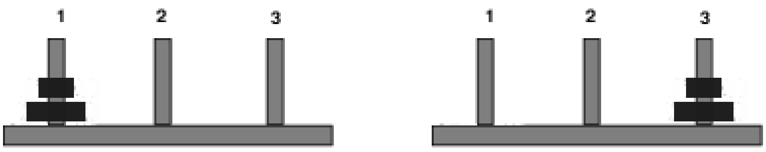
   H: Marcus trys to assassinate Cæsar

   (a) Translate the above sentences into first-order logic.

   (b) Prove that Marcus hates Cæsar using resolution.

7. **Towers of Hanoi**

   Consider the problem of Towers of Hanoi as shown in the figure below. Each disk must be on one of the three pegs and several disks can be on the same peg as long as no disk is on a disk of smaller size. A disk can be moved to another peg if it is clear of all disks and all the disks on the destination peg are larger than the disk you are trying to move.



   Label the small disk A and the large disk B. The labels of the pegs are their numbers. The objective is to move the discs from peg 1 as shown in the left figure to peg 3 as shown in the right figure. Solve this problem with a partial-order planner using STRIPS operators.

   (a) Describe the initial and goal state for this problem. You will also need to choose an appropraite vocabulary to describe the problem.

   (b) Describe the following three operators as STRIPS operators:
   (α): **move-d-d-p**: Move a disk from a disk to a peg
   (β): **move-d-p-p**: Move a disk from a peg to a peg
   (γ): **move-d-p-d**: Move a disk from a peg to a disk

   (c) Show how a partial-order planner would generate a plan for this problem using the graph representation.