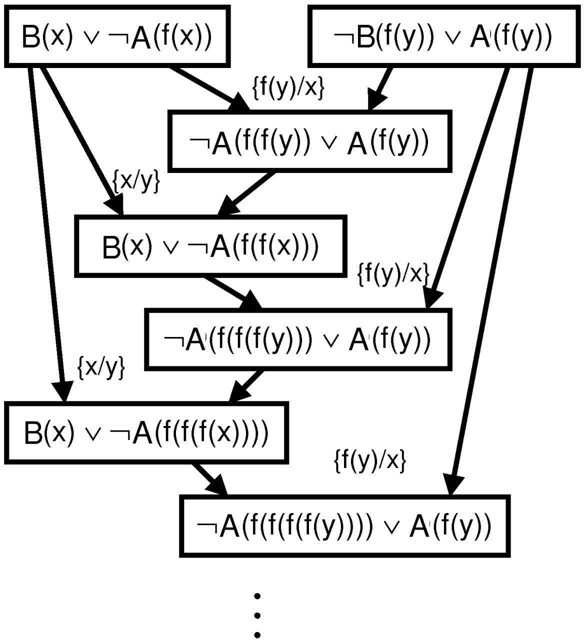
Solutions for CM0312 Coursework II (2009/10)

1. Because of sentence **C** the knowledge-base does not only contain definite clauses (clauses with exactly one positive literal). Hence, neither forward chaining nor backward chaining can succeed. Resolution is guaranteed to find a prove if there is one for any first-order logic knowledge base (but if there is none may not terminate), and hence is the method that should be used here.

[2]

2.
   - **A** : $\neg A(f(x)) \vee B(x)$
     $\neg$**B** : $A(f(y)) \vee \neg B(f(y))$ [2]
   - To show **A** $\models$ **B**, find contradiction in **A**, $\neg$**B**:



   This generates an infinite set of resolvents and the resolution algorithm does not stop. [3]

   - This is an example for a sentence for which no proof exists and shows that first-order logic is only semi-decidable: if a proof exists, we can find one, but according to Gödel's incompleteness theorem not every sentence can be shown to be true or false. [1]

3. If Φ does not contain any function symbols the problem of whether Φ contains a contradiction or not is decidable, i.e. there exists an algorithm (resolution) to find a proof or determine that there is no proof.

   A simple way to prove this is to show that Φ has a finite Herbrand universe. The Herbrand universe is the set of ground terms generated over Φ, and as Φ contains no function symbols (and is finite) this set is finite (no recrusive expressions such as those appearing in Q2 are possible). That means resolution only has to try a finite number of terms and therefore terminates.

   Informal: the propositionalisation of Φ with all constants used in Φ is finite and hence propositional resolution can be used, which is complete. This potentially leaves some variables to deal with, but unification can handle the matching of them without infintie loops and there is no need to propositionalise over infinite domains.

[2]